Show all work.

1. **(6 points)** For each of the following yes/no questions, include a brief justification of your answer.

   (a) **(3 points)** Is the set of vectors $\{(x, y, z)^T | x + y + z = 1\}$ a subspace of $\mathbb{R}^3$? Why or why not?

   (b) **(3 points)** Is the set of polynomials f of degree 4 or less such that $f(7) = 0$ a subspace of P_4? Why or why not?

2. **(11 points)** Answer the following related questions.

   (a) **(6 points)** Find a basis for the nullspace of $\begin{pmatrix} 1 & 3 & -4 & 4 \\ 2 & 0 & 1 & -1 \\ 4 & 1 & 2 & 4 \end{pmatrix}$.

   (b) **(2 points)** What is the dimension of the nullspace above, and what common named vector space is it a subspace of?

   (c) **(3 points)** Is the set of vectors $\left\{ \begin{pmatrix} 1 \\ 3 \\ -4 \\ 4 \end{pmatrix}, \begin{pmatrix} 2 \\ 0 \\ 1 \\ -1 \end{pmatrix} \begin{pmatrix} 4 \\ 1 \\ 2 \\ 4 \end{pmatrix} \right\}$ linearly independent? Why or why not?.

3. **(3 points)** Considered as elements of the vector space of all polynomials on x, show that the set of polynomials $\{x(x-1)(x-2), x(x-1)(x-3), x(x-2)(x-3), (x-1)(x-2)(x-3)\}$ is linearly independent. (Hint: the zero polynomial is the only one which evaluates to zero for each input value x)