

Show all work.

1. **(4 points)** Find a basis for the column space of the matrix  $A = \begin{pmatrix} 1 & 2 & -3 & 2 \\ 0 & 3 & 4 & -5 \\ 3 & 3 & -13 & 11 \end{pmatrix}$ .
  
  
  
  
  
  
  
  
  
  
2. **(6 points)** For each of the following maps  $L$  from one vector space to another, determine whether  $L$  is a linear transformation or not, and state your reasoning.
  - (a)  $L : P_3 \rightarrow \mathbb{R}^2$  where  $L(f(x)) = \begin{pmatrix} f(0) \\ f(3) \end{pmatrix}$ .
  
  
  
  
  
  
  
  
  
  
  - (b)  $L : \mathbb{R}^2 \rightarrow \mathbb{R}^4$  where  $L\left(\begin{pmatrix} x \\ y \end{pmatrix}\right) = \begin{pmatrix} x \\ 2x \\ 0 \\ 3x \end{pmatrix}$ .
  
  
  
  
  
  
  
  
  
  
  - (c)  $L : \mathbb{R}^3 \rightarrow \mathbb{P}_3$  where  $L\left(\begin{pmatrix} r_1 \\ r_2 \\ r_3 \end{pmatrix}\right) = (x - r_1)(x - r_2)(x - r_3)$ .
  
  
  
  
  
  
  
  
  
  
3. **(4 points)** Determine (with brief explanation) the kernel and range of the linear transformation on  $P_4$  given by  $L(f(x)) = f''(x)$ .
  
  
  
  
  
  
  
  
  
  
4. **(6 points)** Let  $L$  be a linear transformation on  $P_2$  given by  $L(f(x)) = xf'(x) + f(2)$ . Find a matrix  $A$  representing  $L$  with respect to the standard basis  $[1, x, x^2]$ .