

Show all work.

1. **(6 points)** Let  $L$  be a linear transformation on  $\mathbb{R}^2$  given by  $L(\mathbf{x}) = (x_1 - x_2, 2x_1 + x_2)^T$ .
  - (a) **(2 points)** What is the matrix  $M$  representing  $L$  with respect to the standard basis  $[\mathbf{e}_1, \mathbf{e}_2]$ ?
  
  
  
  
  
  
  
  
  
  
  - (b) **(4 points)** What is the matrix  $N$  representing  $L$  with respect to the nonstandard basis  $[\mathbf{u}_1, \mathbf{u}_2]$ , where  $\mathbf{u}_1 = (1, 1)^T$  and  $\mathbf{u}_2 = (-1, 0)^T$ ?
  
  
  
  
  
  
  
  
  
  
2. **(4 points)** What is the closest point to  $(1, 3, 6)$  on the plane  $2x - 3y + 3z = 0$ ?
  
  
  
  
  
  
  
  
  
  
3. **(3 points)** What is the closest point to  $(2, 0, 0)$  on the line which passes through both  $(0, 0, 0)$  and  $(-1, 3, 4)$ ?
  
  
  
  
  
  
  
  
  
  
4. **(7 points)** Let  $U$  be the subspace of  $\mathbb{R}^4$  spanned by the vectors  $(1, 3, 2, 0)^T$ ,  $(2, 5, 0, 2)^T$ , and  $(4, 11, 4, 2)^T$ . Find a basis for  $U^\perp$  and state its dimension.