1. The following questions build towards a future proof technique.

   (a) Write a complete proof of the following statement: if n is an even integer, then $3n^2 + n - 5$ is odd.

   (b) Write a complete proof of the following statement: if n is an odd integer, then $3n^2 + n - 5$ is odd.

   (c) What true statement would adequately combine the two results above?

2. In preparation for the next few questions, recall that the logical implication operator can be defined entirely in terms of disjunctions and negations: $P \to Q \equiv \neg P \vee Q$. In fact, *any* logical expression you might wish can be built using conjunctions, disjunctions, and negations. For instance, consider the mysterious function of P, Q, and R with the truth table given below:

P	Q	R	$f(P,Q,R)$
F	F	F	F
F	F	T	F
F	T	F	T
F	T	T	F
T	F	F	T
T	F	T	T
T	T	F	F
T	T	T	F

   We could write that simply as a disjunction of 3 conjunctions corresponding to each of the places where it's true, so

$$f(P,Q,R) \equiv (\neg P \wedge Q \wedge \neg R) \vee (P \wedge \neg Q \wedge \neg R) \vee (P \wedge \neg Q \wedge R)$$

   This is not the most efficient way to do this, but it's an easy way to demonstrate that conjunctions, disjunctions, and negations are all you will ever truly need to write a logical expression. The following questions address the possibility that you might not even need all three of these operations.

   (a) Is it possible to write the conjunction operation ($P \wedge Q$) entirely in terms of disjunctions and negations? Either show how or explain why not.

   (b) Is it possible to write the disjunction operation ($P \vee Q$) entirely in terms of conjunctions and negations? Either show how or explain why not.

   (c) Is it possible to write the negation operation ($\neg P$) entirely in terms of conjunctions and disjunctions? Either show how or explain why not.

   (d) **BONUS:** It is impossible to write every single possible logical statement in terms of only *one* of the standard operations (negations, conjunctions, disjunctions, or implications). However, if we consider nonstandard binary operations, there are 16 different possible ways to fill in a truth table, and maybe one of them will suffice. Note that these 16 ways include some pretty silly operations like "always result in true" or "negate the left operand and ignore the right", but maybe some of the others will work. Below, show a truth table for some nonstandard operation $\otimes$, and show how conjunction, disjunction, and negation could all be rewritten in terms of $\otimes$.

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