1. In calculus, you probably encountered the following formal definition of a limit: $\lim_{x\to a} f(x) = L$ means that for every positive real ϵ there is a positive real δ such that for any x, if $0 < |x-a| < \delta$, then $|f(x) - L| < \epsilon$.

   (a) Write this definition symbolically.

   (b) Quantify the negation of this statement: come up with a quantified, easy-to-use statement of what it means to say that $\lim_{x\to a} f(x) \neq L$.

   (c) Use the results of the previous part to come up with an argument that $\lim_{x\to 3} x^2 \neq 5$.

2. For each of the following statements, either demonstrate that it is false with a counterexample or that it is true with a formal proof.

   (a) For all integers a, b, c, and d and natural number n, if $a \equiv b \pmod{n}$ and $c \equiv d \pmod{n}$, then $ac \equiv bd \pmod{n}$.

   (b) For all integers a, b, and c, if $a \mid (b-1)$ and $a \mid (c-1)$, then $a \mid (bc-1)$.

   (c) For all integers a, b, and c, if $(ab) \mid c$ then $a \mid c$.

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