

At this point your proofs might make use of several possible techniques: direct proof, proof by contrapositive, proof by contradiction, and proof by construction. Prove each of the following statements in whichever way seems to be most effective to you.

1. For any integer n , if 3 does not divide $4n^2 + n$, then 3 does not divide n .
2. There is a triple of integers a , b , and c such that $a^2 + b^2 = c^2$ (called a *Pythagorean triple*) such that $c = 5$ and a and b are consecutive integers.
3. For any real x , it is the case that $x(1 - x) \leq \frac{1}{4}$.
4. For all integers a and b , $(a + b)^3 \equiv (a^3 + b^3) \pmod{3}$.
5. For real α and x , if α is irrational and x is rational, then the product αx is irrational.