1. Briefly explain whether each of the following very similar-looking statements is true or false, and why. Are any of these statements saying identical things, or do they all have a different meaning?

   (a) $(\exists m \in \mathbb{N})(\exists n \in \mathbb{N})(m > n)$.

   (b) $(\exists n \in \mathbb{N})(\exists m \in \mathbb{N})(m > n)$.

   (c) $(\exists m \in \mathbb{N})(\forall n \in \mathbb{N})(m > n)$.

   (d) $(\exists n \in \mathbb{N})(\forall m \in \mathbb{N})(m > n)$.

   (e) $(\forall m \in \mathbb{N})(\exists n \in \mathbb{N})(m > n)$.

   (f) $(\forall n \in \mathbb{N})(\exists m \in \mathbb{N})(m > n)$.

   (g) $(\forall m \in \mathbb{N})(\forall n \in \mathbb{Z})(m > n)$.

   (h) $(\forall n \in \mathbb{N})(\forall m \in \mathbb{Z})(m > n)$.

2. A function f (with domain and range $\mathbb{R}$) is said to be *strictly increasing* when, for all real numbers x and y with $x < y$, $f(x) < f(y)$.

   (a) Write the definition of a strictly increasing function as a quantified statement with symbols only.

   (b) Write the definition of a function which is *not* strictly increasing (i.e. the negation of the above) as a quantified statement with symbols only.

   (c) Using the previous result, how could we, for instance, argue that $f(x) = x^3 - 9x$ is not strictly increasing?

3. Consider the open sentence in x: $(\exists t \in \mathbb{Z})(t \cdot x = 20)$.

   (a) Is the resulting statement when $x = 8$ true or false?

   (b) Is the resulting statement when $x = -2$ true or false?

   (c) What is the truth set of this sentence, i.e. the set of values x which make it true?