For each of the following conjectures, test if it appears to be true or false with some examples. Once you are reasonably satisfied that you have a good idea as to whether it is true or not, do one of the two following as appropriate:

- If true, work out the logical flow for a proof, identifying known facts, desired conculsions, and determining how one might lead to the other. If time permits, write the formal proof itself.
- If false, give a counterexample. Then check to see if there are interesting limitations or modifications of this statement which might be true.

1. For all integers a, b, and c, if $a \mid b$ and $b \mid c$, then $a \mid c$.
2. For all integers a, b, and c, if $a \mid b$ and $a \mid c$, then $a \mid b + c$.
3. For all integers a, b, and c, if $a \mid c$ and $b \mid c$, then $a + b \mid c$.
4. For all integers a, b, and c, if $a \mid b$ and $a \mid c$, then $a \mid bc$.
5. For all integers a, b, and c, if $a \mid c$ and $b \mid c$, then $b \mid c$.
6. For all integers a and b, if $a \mid b$, then $a^2 \mid b^2$.
7. For all integers a and b, if $a \nmid b$ and $a \nmid c$, then $a \nmid bc$.
8. For any integer n, if n is odd, then 8 divides $n^4 + 4n^2 + 11$.
9. For all real numbers x and y, $\sqrt{xy} \leq \frac{x+y}{2}$.
10. For every integer a, if $a \equiv 2 \pmod 8$, then $a^2 \equiv 4 \pmod 8$.
11. For all integers a, b, and c and natural numbers n, if $a \equiv b \pmod n$ and $b \equiv c \pmod n$, then $a \equiv c \pmod n$.