1. Evaluate the following proof: is there a failure of logic or construction?

   Proposition 1. *For an integer m, if m is odd, then $m+6$ is also odd.*

   Proof. For $m+6$ to be an odd integer, there must exist an integer n such that

   $$m + 6 = 2n + 1.$$

   By subtracting 6 from both sides of this equation, we obtain

   $$\begin{aligned} m &= 2n - 6 + 1 \\ &= 2(n-3) + 1. \end{aligned}$$

   By the closure properties of the integers, $(n-3)$ is an integer, and hence, the last equation implies that m is an odd integer. This proves that if m is an odd integer, then $m-6$ is an odd integer. □

2. For each of the following true statements, we're going to consider its contrapositive, which is a logically equivalent statement. Then we're going to prove it.

   (a) For any integer n, if n^3 is even, then n is even.

   (b) For any integers m and n, if mn is even, then m or n must be even.

   (c) For nonnegative real numbers a and b, if $\sqrt{ab} \neq \frac{a+b}{2}$, then $a \neq b$.

   (d) For a right triangle with legs of positive length a and b and hypotenuse of length c (note that $a^2 + b^2 = c^2$), if the triangle has area $\frac{1}{4}c^2$, then the triangle is isosceles (having legs of equal length).

3. A followup to the last part of the above: for the same conditions, we can also prove that if the triangle is isosceles, then its area is $\frac{1}{4}c^2$. What conclusion encompasses both of these true facts?