1. For each of the following *biconditional* statements, we'll break it up into two conditionals, assess their truth value, and prove the strongest thing we can about it.

   (a) For any integer n, it is the case that n is even if and only if n^2 is divisible by 4.

   (b) For any integer a, it is the case that $a \equiv 3 \pmod 7$ if and only if $a^2 + 5a \equiv 3 \pmod 7$ is even.

   (c) For any positive real number x, the quantity $\sqrt{x}$ is irrational if and only if x is irrational.

   (d) For any integer a, a is odd if and only if there are integers x and y such that $ax + 2y = 1$.

2. For each of the following *existence* statements, we'll look at whether it appears to be true and try to build an argument about its truth.

   (a) There exist integers x and y such that $4x + 6y = 2$.

   (b) There are integers x and y such that $6x + 15y = 2$.

   (c) There exist integers x and y such that $6x + 15y = 9$.

   (d) There exists a real number x such that $x^3 - 4x^2 = 7$.

   (e) For any rational numbers p and q, if $p < q$, then there exists a rational number x such that $p < x < q$.