

The following true statements are amenable to a casewise decomposition (possibly with “WLOG”, or after a contrapositive rephrasing). Let’s figure out what sort of casewise decomposition works for each, and then prove them.

1. For any integer u , if there is an integer solution to $x^2 + x - u = 0$, then u is even.
2. For any real number x , $x^2 \geq 0$ (we’ve assumed this one previously, but let’s go ahead and prove it).
3. For any integers a , b , and c , if either $c \mid a$ or $c \mid b$, then $c \mid ab$.
4. For all real numbers x and y , $|x - y| \geq |x| - |y|$.
5. There are no natural numbers a and n such that $n \geq 2$ and $a^2 + 1 = 2^n$.