1. **(11 points)** The statement "for any real number x, if x is not an integer, then $7x$ is also not an integer" is false. State its converse. Is the converse true or not? Briefly explain your reasoning.

2. **(18 points)** Fill in the truth table for each of the following statements, and identify the statement as a tautology, a contradiction, or neither.

   (a) $(P \vee Q) \leftrightarrow P$.

P	Q	

   (b) $[(P \to Q) \wedge \sim Q] \to \sim P$.

P	Q	

3. **(22 points)** Prove that for any real number x, if $x^3 + x < 10$, then $x < 2$.

4. **(22 points)** Prove that for any integers m and n, if n is odd, then $nm - m^2$ is even.

5. **(20 points)** Prove that for any positive integer n, it is the case that

$$1 \cdot 2^1 + 2 \cdot 2^2 + 3 \cdot 2^3 + 4 \cdot 2^4 + \cdots + n \cdot 2^n = (n-1)2^{n+1} + 2$$

6. **(20 points)** Prove that there is no integer n such that $n \equiv 4 \pmod 6$ and $n \equiv 2 \pmod 9$.